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STRAINS

Ultimate Limit State Design of three-dimensional reinforced concrete structures: a numerical approach

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Computational Modelling of concrete and concrete Structures

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Bad Hofgastein, Austria

Introduction

- Evaluate the ultimate **load bearing capacity** of massive (**3D**) **reinforced concrete structures**
- Cannot be modelled as 1D (beams) or 2D (plates) structural members
- Based on the **yield design (or limit analysis) approach**

Outline

- Mechanical model
  - Modelling concrete
  - Modelling reinforced concrete
- Yield Design – Limit Analysis
  - Static approach
  - Kinematic approach
- Numerical implementation of both static and kinematic approaches
- Practical example

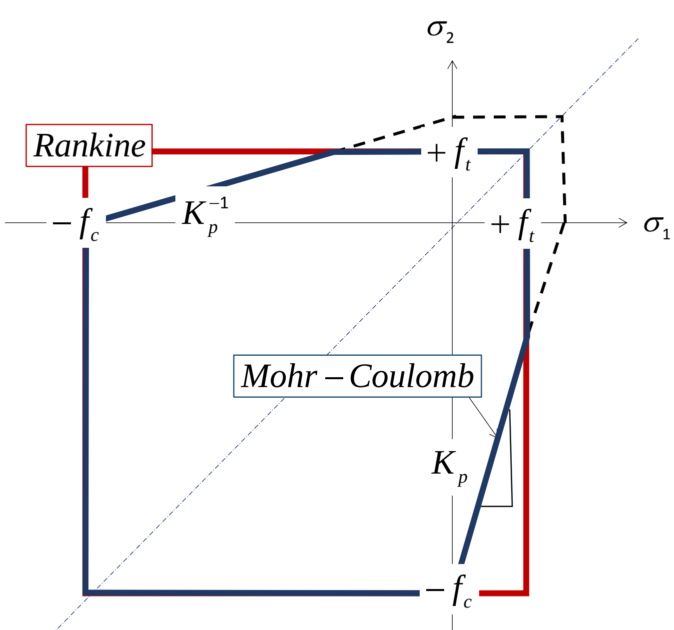
Hugues Vincent

Modelling concrete

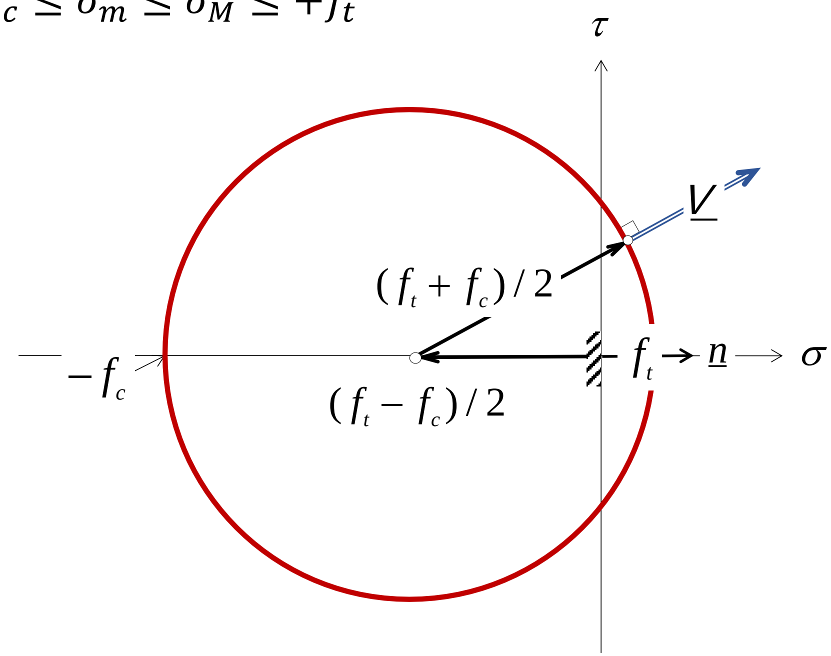
- **Mohr-Coulomb** criterion with a tension cut-off [Drucker, 1969] [Chen, 1969]

$$F^c(\underline{\underline{\sigma}}) = \sup\{K_p\sigma_M - \sigma_m - f_c; \sigma_M - f_t\} \leq 0 \qquad K_p=(1+\sin\varphi)/(1-\sin\varphi)$$

- Much simpler **Rankine** criterion $F^c(\underline{\underline{\sigma}}) \leq 0 \Leftrightarrow -f_c \leq \sigma_m \leq \sigma_M \leq +f_t$

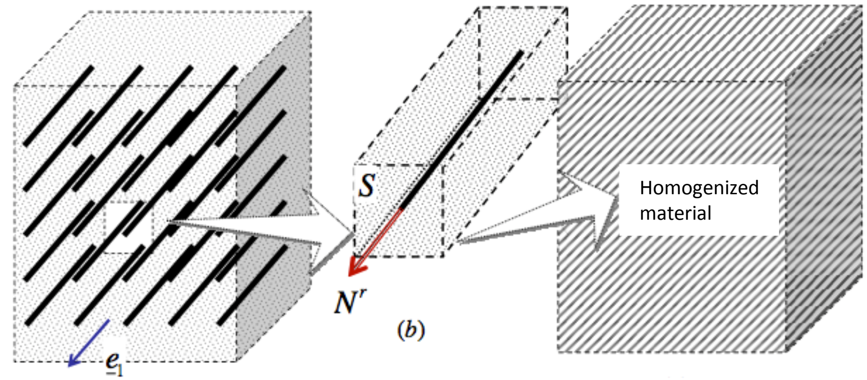


Moh-Coulomb and Rankine criteria under plane stress conditions



Geometrical representation of the Rankine criterion in the Mohr-plane

Modelling reinforced concrete



- **Periodic reinforcement**:
  - ➢ Replace concrete and reinforcement by a **homogenized material**
  - ➢ Macroscopic strength condition

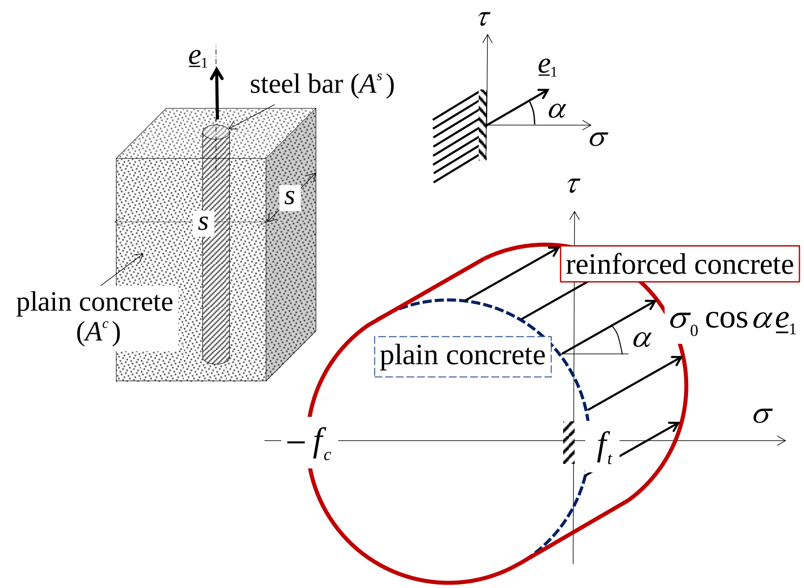
    [de Buhan and Taliercio, 1991]

    [de Buhan, Bleyer, Hassen, 2017]
  - ➢ Tensile resistance of the rebars

$$F^{rc}(\underline{\underline{\sigma}}) \leq 0$$

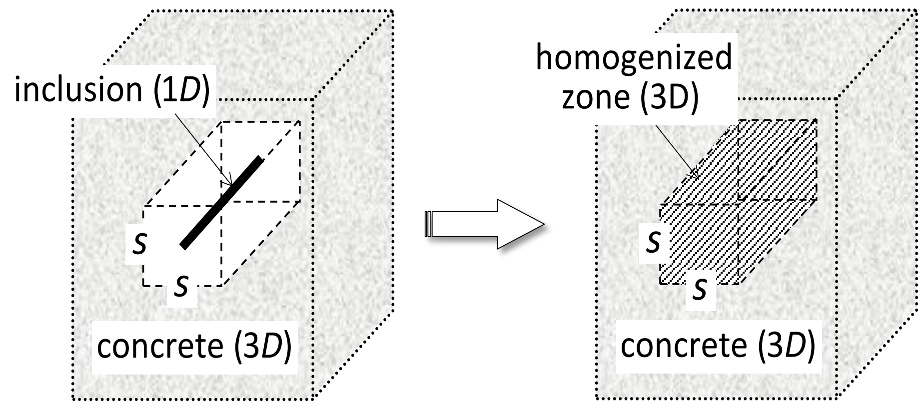
$$\Leftrightarrow \begin{cases} \underline{\underline{\sigma}} = \underline{\underline{\sigma}}^c + \sigma^r \underline{e}_1 \otimes \underline{e}_1 \\ with \quad F^c(\underline{\underline{\sigma}}^c) \leq 0 \quad and \quad -k\sigma_0 \leq \sigma^r \leq \sigma_0 \end{cases}$$

$$\sigma_0 = \frac{A^s f_y^s}{s^2} = \eta f_y^s$$

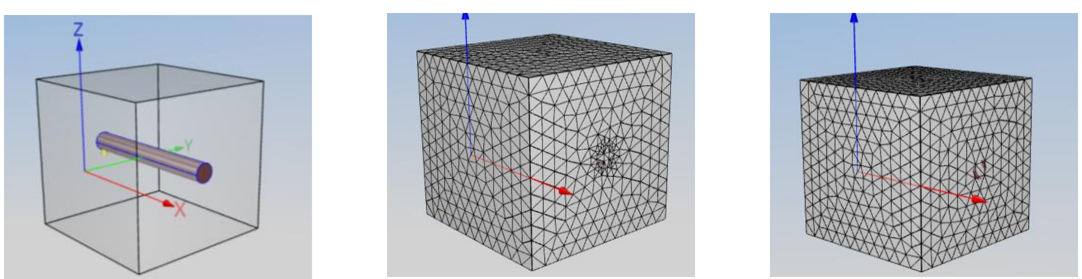


Modelling reinforced concrete

- **Isolated rebar**:
    - ➢ 1D-3D mixed modelling approach generates stress singularities
    - ➢ Each rebar modelled as 3D volume body
    - ➢ Homogenization procedure [Figueiredo, MS, 2013]



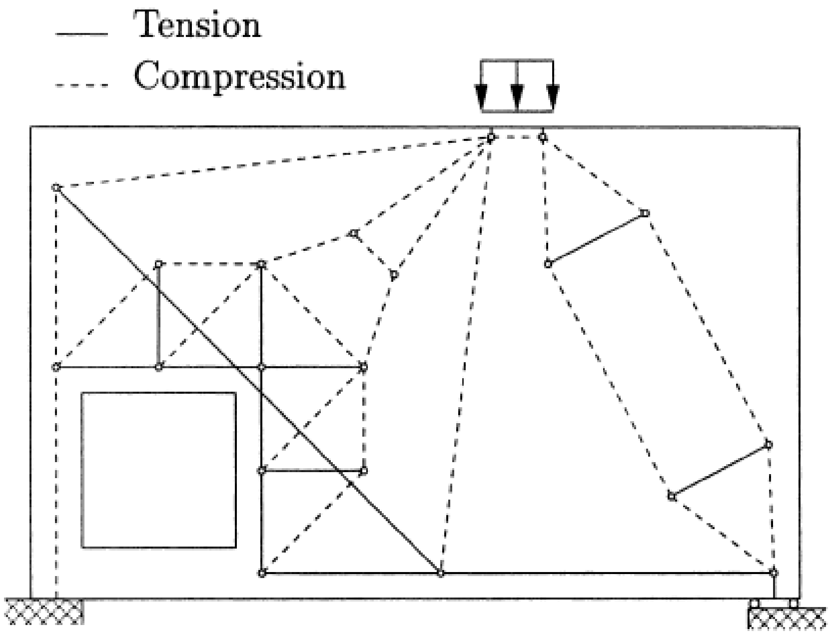
- Homogenized zone larger than the inclusion
- Numerically cheaper
- Control the size of the homogenized zone



Yield design – limit analysis

[Drucker,1952] [Chen, 1982][Salençon, 1983] [Hill, 1950]

- Find the **Ultimate Limit State** of a structure
- Without performing a step-by-step elasto-plastic analysis
- Two separate calculations :
  - Static calculation (**lower bound**)
  - Kinematic calculation (**upper bound**)
- Estimation of the **capacity** of the structure with an **error estimator** for the FE model



[source: Poulsen, IJSS, 2000]

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Numerical implementation of the lower bound static approach

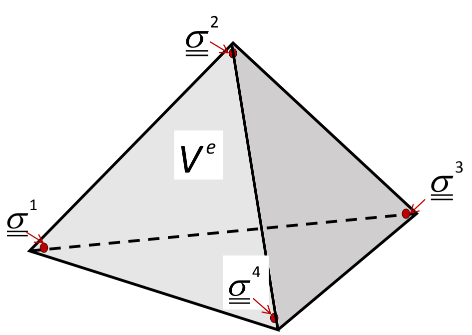
$$Q^+ = \sup\left\{Q; \exists \underline{\underline{\sigma}}\ S.A\ Q, F(\underline{\underline{\sigma}}(\underline{x})) \leq 0\ \forall \underline{x}\right\}$$

- **Statically Admissible** stress field:
  - Respects equilibrium at any point in the structure
  - Continuity of the stress-vector across possible stress jump surface
  - Boundary conditions

- Respect **strength conditions**

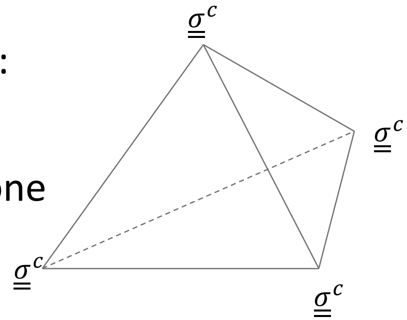
$$F^c(\underline{\underline{\sigma}}(\underline{x})) \leq 0\ \forall \underline{x} \in V^c \ \ and \ \ F^{rc}(\underline{\underline{\sigma}}(\underline{x})) \leq 0\ \forall \underline{x} \in V^{rc},\ \ V = V^c \cap V^{rc}$$

- **Finite element** method
  - Tetrahedral FE
  - Linear variation of the stress field
  - Stress jump across adjacent elements

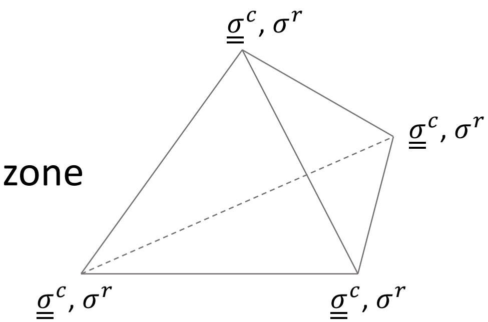


Numerical implementation of the lower bound static approach

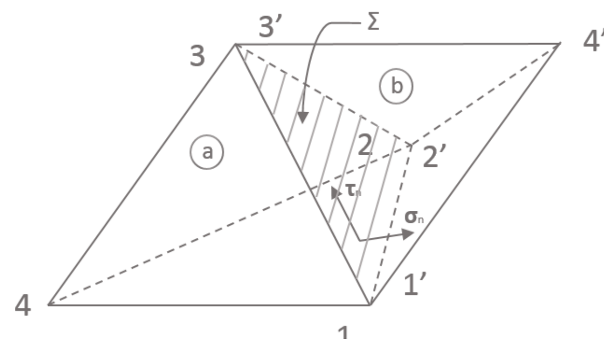
- Variables at each node:



  - Plain concrete zone
  - Reinforced concrete zone



- Linear constraints on the stress variables to express:
  - Equilibrium
  - Continuity of the stress vector across adjacent elements
  - Boundary conditions



- FE implementation of the **lower bound static approach** of yield design translated into a maximisation problem (**Semidefinite Programming**) solved with Mosek:

$$Q^{+} \geq Q^{lb} = \underset{\{\Sigma\}}{Max}\, Q = {}^{T}\{A\}\{\Sigma\} \text{ subject to } \begin{cases} [B]\{\Sigma\} = \{C\} & \text{equilibrium} \\ F(\{\Sigma\}) \leq 0 & \text{strength criteria} \end{cases}$$

Numerical implementation of the upper bound kinematic approach

- **Dualization** of the lower bound:
  - given any kinematically admissible (K.A.) velocity field $\underline{U}$, the so-called **maximum resisting work** is:

$$P_{mr}(\underline{U}) = \int_{\Omega^c} \pi^c(\underline{\underline{d}})\, d\Omega^c + \int_{\Omega^{rc}} \pi^{rc}(\underline{\underline{d}})\, d\Omega^{rc} + \int_{\Sigma^c} \pi^c(\underline{n}; \underline{V})\, d\Sigma^c + \int_{\Sigma^{rc}} \pi^{rc}(\underline{n}; \underline{V})\, d\Sigma^{rc}$$

- **Support functions** defined as:

$$\pi^{c/rc}(\underline{\underline{d}}) = \sup\{\underline{\underline{\sigma}} : \underline{\underline{d}};\, F^{c/rc}(\underline{\underline{\sigma}}) \leq 0\}$$
$$\pi^{c/rc}(\underline{n}; \underline{V}) = \sup\{(\underline{\underline{\sigma}}.\underline{n}).\underline{V};\, F^{c/rc}(\underline{\underline{\sigma}}) \leq 0\}$$

- The **ultimate load** must satisfy the following inequality, valid for any K.A. velocity field U:

$$P_{ext} \leq P_{mr}$$

Numerical implementation of the upper bound kinematic approach

- **Finite element** method
  - Tetrahedral FE
  - Quadratic variation of the velocity field
  - Velocity jump across adjacent elements

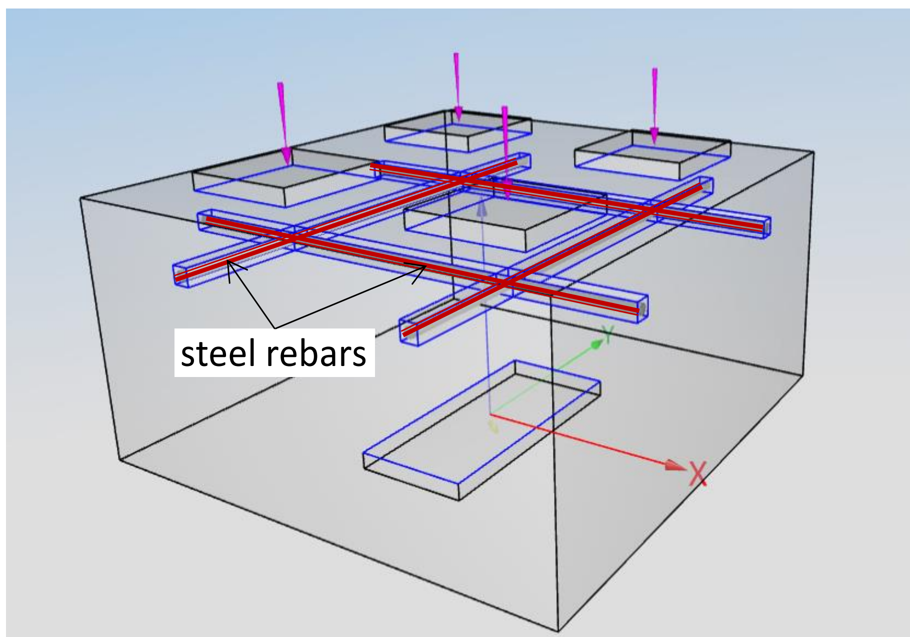
$$Q^{+} \leq Q^{ub} = \underset{\{U\}}{Min}\{P_{mr}(\{d\},\{V\})\}$$

$$subject\ to \begin{cases} \{d\} = [D]\{U\} \\ \{V\} = [E]\{U\} \\ {}^{T}\{F\}\{U\} = 1 \end{cases}$$

- Both approaches presented as **maximization** or **minimisation** problems: treated by means of **Semi-definite programming** (SDP)

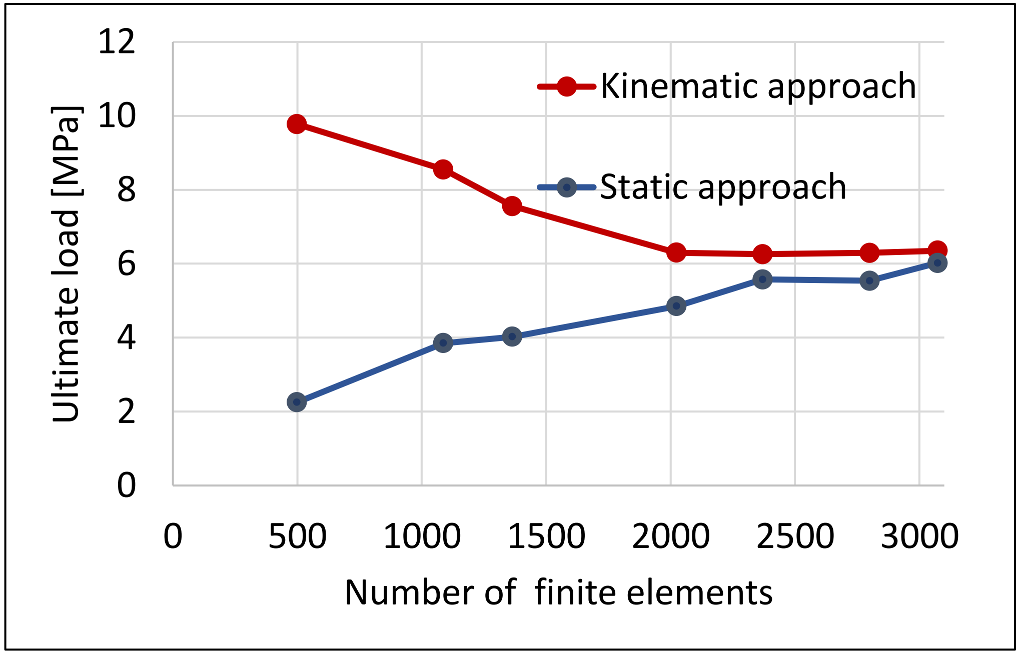
Failure design of a bridge pier cap

- Truly massive three dimensional structure
- 3x3x1.5 m^3 parallelepipedic concrete block
- Uniform pressure on top of four square pads
- Rigid connection on a 1.5x0.7 m^2 rectangular area placed at the centre of the bottom surface



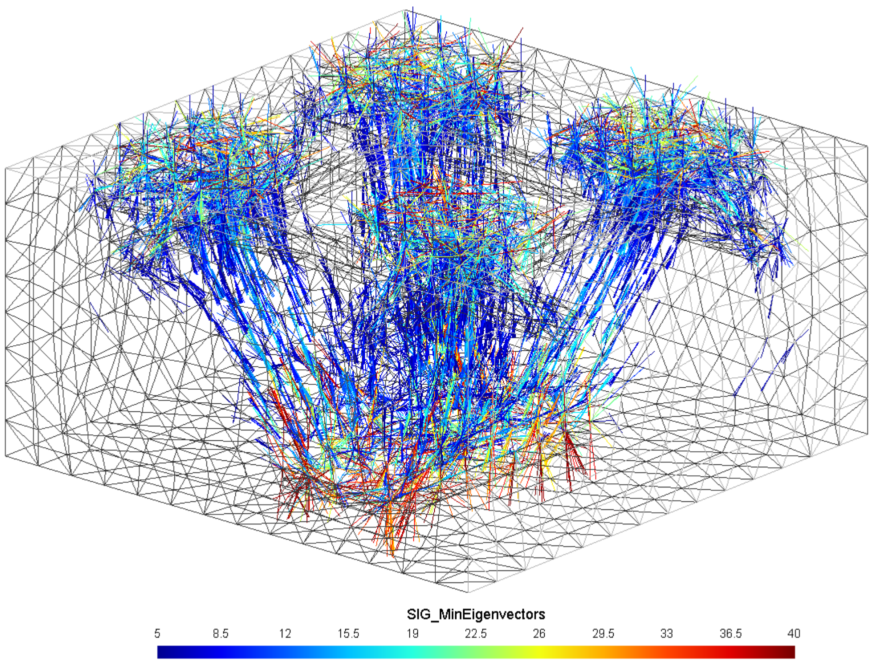
Failure design of a bridge pier cap

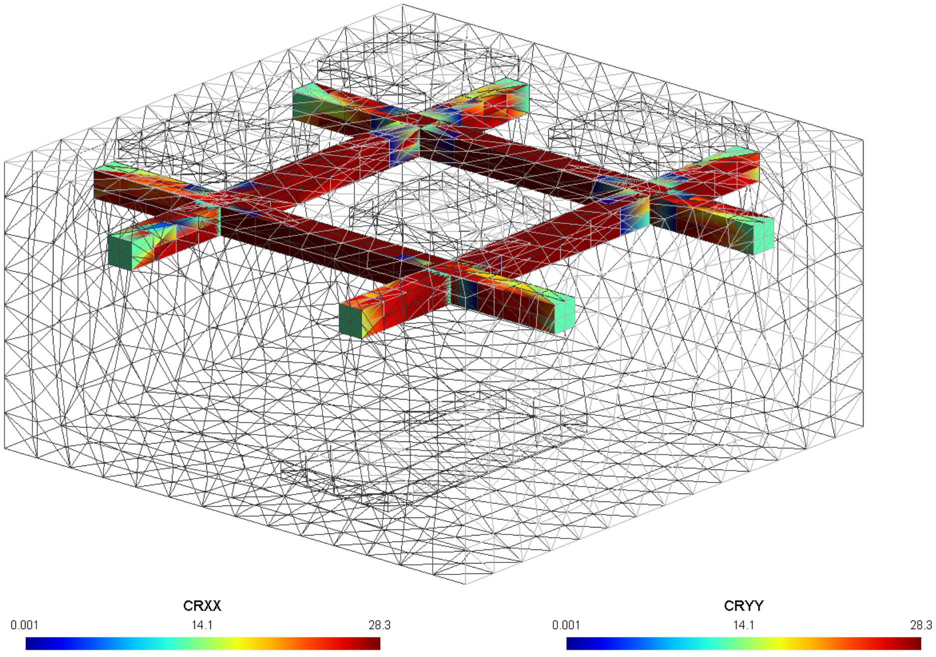
- Static lower bound approach
- Kinematic upper bound approach
- Several numerical analyses performed
- Convergence of both approaches



Static approach - results

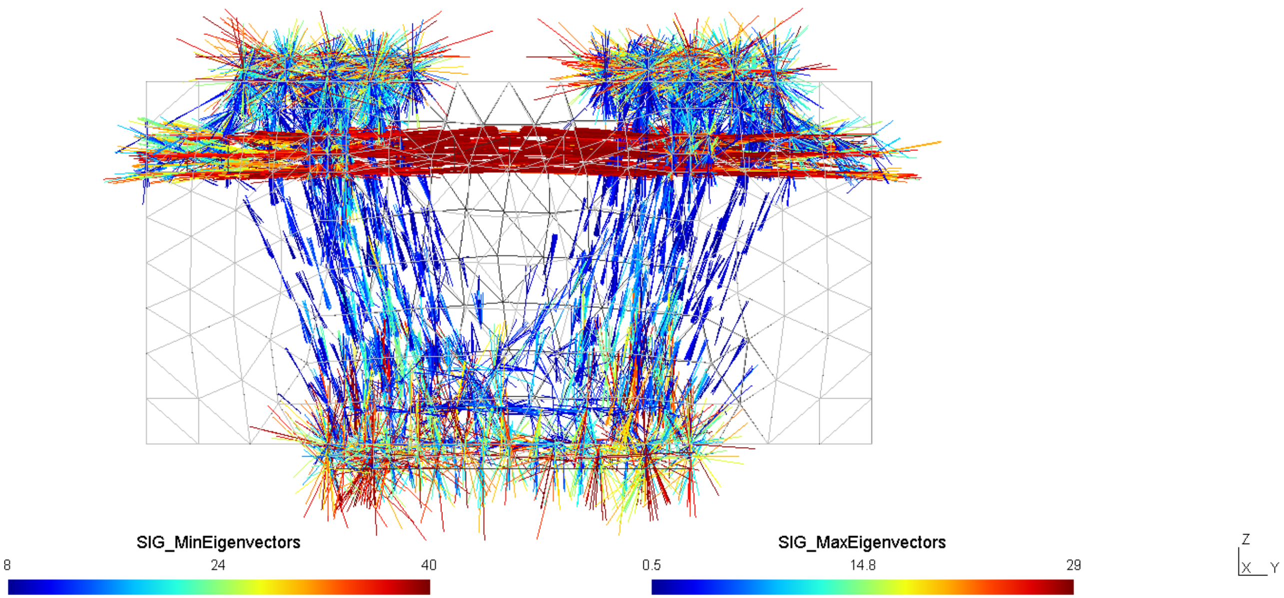
- **Principal compressive stresses** in plain concrete
- **Tensile stresses** in the homogenized reinforcement





Static approach - results

- **6.02 MPa** on each of the four loading pads (unreinforced: 3.12 Mpa)
- gives a clear intuition of the **optimized stress field** equilibrating the applied loading
  - Compressive stresses (**struts**)
  - Tensile stresses (**ties**)



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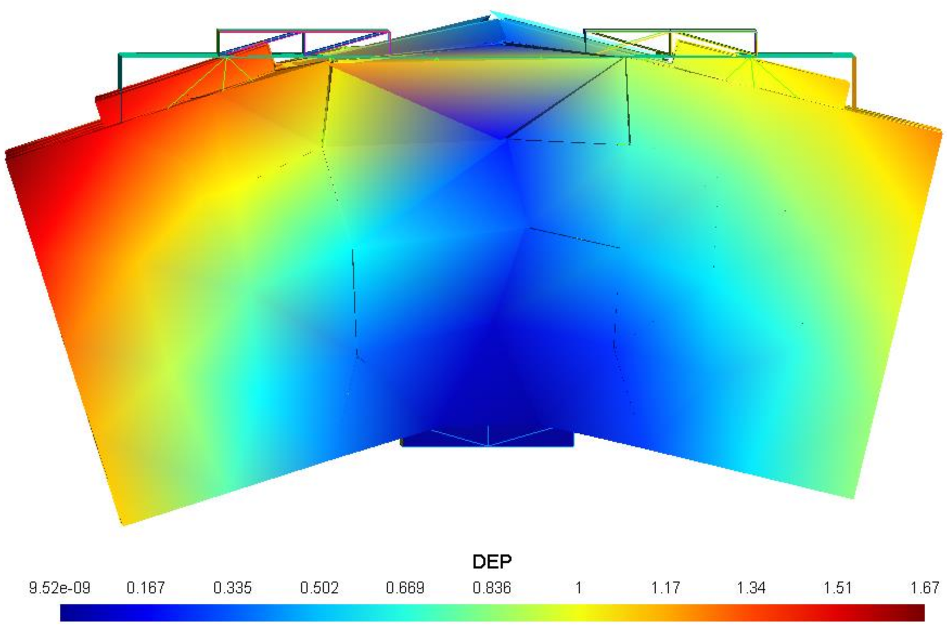
Kinematic approach - results

- Failure **mechanism**
- **6.35 MPa** on each of the four loading pads (unreinforced: 3.68 Mpa)
- 2.5 % error

$$6.02\ MPa \leq Q^{+} \leq 6.35\ MPa$$



Conclusion

- Dedicated FE computed code developed
  - Gives the **Ultimate load bearing capacity** of 3D reinforced concrete structures
  - **Yield design** approach
  - Gives rigorous lower bound (i.e. **conservative**) and upper bound (error estimator)

- Relies on two decisive steps:
  - **Homogenization**-inspired model for individual reinforcement
  - Optimization problem (using **SDP**)

- Extension: **Remeshing procedure** based on information provided by both approaches (stress and velocity fields)

Thank you

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SDP formulation

- **Mohr-Coulomb criteria** expressed in terms of **principal stresses**
  - Semidefinite Programming optimization problem

$$\begin{cases} t_M \underline{\underline{1}} - \underline{\underline{\sigma}} \succeq 0 \quad and \quad t_m \underline{\underline{1}} - \underline{\underline{\sigma}} \preceq 0 \\ K_p t_M - t_m - f_c = 0 \end{cases}$$

$$\underline{\underline{A}} \succeq 0 \Leftrightarrow \underline{x}.\underline{\underline{A}}.\underline{x} \geq 0 \ \forall \underline{x}$$

- Introducing auxiliary **symmetric matrix variables** $\underline{\underline{X}}$ and $\underline{\underline{Y}}$:

$$\underline{\underline{X}} + \underline{\underline{Y}} + (1 - K_p^{-1}) t_m \underline{\underline{1}} \;=\; K_p^{-1} f_c \underline{\underline{1}}$$
$$with \ \underline{\underline{X}} \succeq 0 \ \ and \ \underline{\underline{Y}} \succeq 0$$

- Similarly, the **Rankine-type cut-off strength criteria**:

$$\sigma_M - f_t \leq 0 \Leftrightarrow \underline{\underline{\sigma}} - f_t \underline{\underline{1}} \preceq 0$$

$$\underline{\underline{\sigma}} + \underline{\underline{Z}} - f_t \underline{\underline{1}} = 0 \ \ and \ \underline{\underline{Z}} \succeq 0$$